Design and Implementation of an Autonomous Underwater Vehicle: Yellowfinn II

Stephen Cronin, Robert Goring, Zachary Joshwick, Kayla Ormiston, Lingxiao Wang, Evan Williams

***Abstract*—The Robosub team of the Robotics Association at Embry-Riddle discuss their entry into the 20th AUVSI Robosub competition. The overall design methodology “autonomy happens” is what embodies this vehicle, were core capabilities of the vehicle are valued over just being able to complete a task. To achieve this design philosophy, a new vehicle chassis, electronics subsystem and software backbone were designed and the design decisions made therein will be discussed highlighting the process taken to arrive at this entry.**

I. Introduction

This competition marks the twentieth anniversary of the AUVSI Robosub competition. This revision of the competition brings about many of the same tasks with thematic changes as well as a revamped manipulation task. This consistent task arrangement allowed the team to pull on past knowledge of how to approach elements such as testing and vehicle designs that would be adverse and to refocus on their efforts on building a platform that aligned with the larger direction of the Robotics Association and of autonomous vehicle design overall.

The design philosophy of the new vehicle is summed up by the mantra “autonomy happens”. What this means is that the focus of the vehicle is not on accomplishing the tasks in question but designing a vehicle to be robustly capable in core operations, that of sensing, control and path planning. When these capabilities are fully realized, the task of determining where to tell the vehicle to drive becomes a trivial operation.

Achieving this design goal required a relook at the chassis of the vehicle as well as the sensing suite onboard. Through the support of the research arm of the Robotics Association, we gained access to the later in the form of an imaging sonar and DVL. To utilize these devices effectively, a new chassis was required. It would not only support the new sensors, but to provide a platform to build the larger autonomy goals of our organization around. As the vehicle is designed to be capable irrelevant of the competition, the vehicle is also intended to be utilized in conjunction with our Maritime RobotX platform, Minion, for both that competition as well as cooperative robotics research.

A solid design goal and plan is nothing without a team to back it up however. This year’s Robosub team is multidisciplinary group comprised of electrical, mechanical and software engineering students. In total 6 students actively worked on this project, twice the number of last year. These students ranged from first year to graduate students, allowing for student’s knowledge to be passed down to newer members, strengthening skill sets and ensuring that subject matter expertise is not lost over time.

II. Hardware Overview

The RoboSub 2017 platform represents a radical deviation from the design methodology of the previous 7 years of Embry-Riddle Aeronautical University RoboSub Platforms. The previous designs have utilized a Pelican case as the main waterproofing element of the system. This case has been attached to a frame that supports the thrusters and other sensors that comprise the system.

This methodology presents a number of key issues, the primary among them being the flex and deformation of the case under pressure. The case, while made of a rigid plastic, deforms significantly under pressure at the operating depths of the competition. This causes a changeover of the buoyancy of the system at a depth of around 6 to 9 feet from positive buoyancy, to negative. From a software perspective this drastically increases the complexity of the control algorithm, and from a system safety perspective this behavior is dangerous and can result in loss of the system if there is a power system or control failure below the buoyancy changeover point.

With the inclusion of an Imaging Sonar, Doppler Velocity Logger, and the goal of creating a collaborative robotics system with the Embry-Riddle RobotX platform, this failure mode was deemed unacceptable and a new platform design was commissioned.

The goal of the new platform was to create a modular platform that can be utilized for both the RoboSub and RobotX competitions in both a standalone and collaborative mode. This use case causes several specific design elements and methodologies to be implemented.

A. Frame

The frame design pulls elements from previous years’ designs and combines them with elements from commercially available platforms to create a lightweight and rigid frame. The frame utilizes bent aluminum crossmembers coupled to rigid side panels. The panels are bolted together to allow the vessel to flat pack for easy transport as a checked bag on a commercial aircraft.

As shown in Figure 1, the side panels, colored yellow, support the rest of the crossmembers colored in blue. This design increases the moment of inertia of the individual crossmembers as compared to a system comprised of just flat bars, increasing the resistance to bending and torsional loads. By changing the direction of the bends, the frame can said loads from any angle, increasing its overall rigidity,

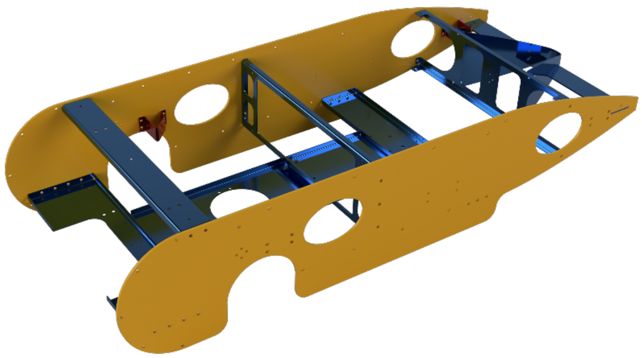

Figure 1: Yellowfinn II Frame CAD

while using the same or less material than other frame options.

B. *Hydrodynamic panels*

Range and efficiency are two factors that bear significant importance when designing a vessel for operation in real world scenario. As such Yellowfinn II has been designed with outer cover panels to increase the hydrodynamic efficiency of the system. By doing so the vessel's form drag decreases significantly, making movement more efficient, and thus increasing the overall efficiency of the system. A cad render of the panels is included shown in Figure 3.

The outer panels also serve to protect the inner components from damage during operation. The smooth outer shape of the vessel resists snagging or catching on external objects such as buoy lines. This methodology is also carried on to the tether and battery access ports which utilize flush mount hinges and latches to ensure that access into the vessel is quick, but does not disturb the smooth outer profile.

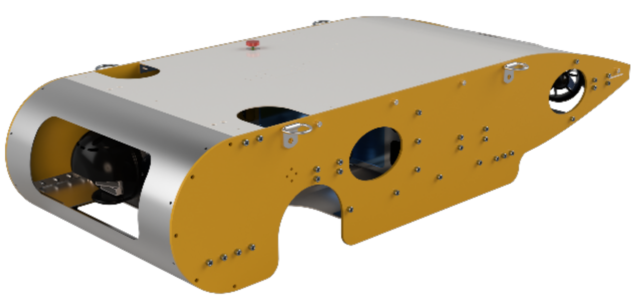

Figure 3: Yellowfinn II with Panels

C. *System layout*

The system's electrical components are housed within a metal computer enclosure pictured below. The computer enclosure is comprised of a 5-inch diameter metal tube and a 14-inch-long by 6-inch-tall aluminum box, show below in Figure 4.

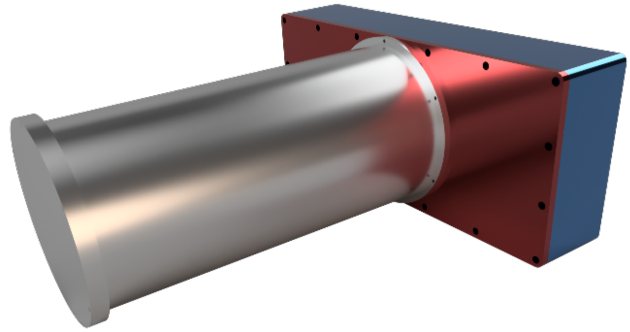

Figure 4: Computer Enclosure

These elements are coupled together with fasteners and sealed with O-rings. This design was chosen due to the number of connections required for system operation and intended buoyancy of the system. The electronics of the system can fit into a tube with an internal diameter of 5 inches, but the area required to fit the number of passthroughs needed for vessel operation is greater than the area available on the end caps of the tube. Rather than increasing the diameter of the tube, the electronics enclosure utilizes a large aluminum block to provide the necessary room to house the electronics. Figure 2 shows the first revision of this block.

The Connector passthrough block also serves as an area providing additional room tohouse the custom electronics

Figure 2: First Revision Connector Passthrough

boards described in the electronics section in a later section.

D. *Thrusters*

The thruster layout for yellowfin 2 is a change from the configuration of previous years. The new configuration utilizes 7 thrusters to provide a full 6 degree of freedom capable platform. This provides numerous benefits, the principal among them being the enhanced controllability and maneuverability. Additionally, the platform is able to more effectively station keep and resist the forces of currents while in operation.

E. *Camera enclosures*

The camera enclosures for Yellowfinn II are also new for the 2017 competition. The enclosures solve a number of issues associated with the previous generations of enclosures. The previous camera enclosures did not have a way of indexing the camera rigidly when assembling. This caused issues when attempting to align and focus the camera, making the whole assembly difficult to use and service. The new camera enclosure is fully machines and has access to both sides of the camera. Additionally, the camera is affixed to an internal bracket that acts as both a heat sink and as a constant mounting point. The bracket is rigidly affixed to the rear cap, allowing for easy installation that has a static reference every time the camera is serviced. Multiple views of the housing are included in Figure 5.

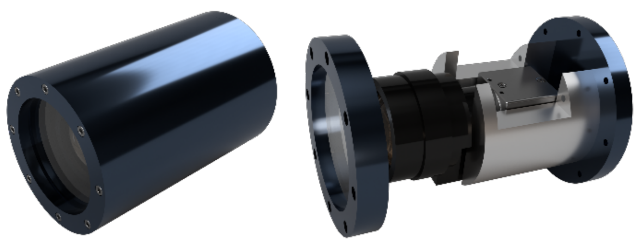

Figure 5: Camera enclosure whole (left) and internal (right)

F. *Mounting and serviceability*

Yellowfinn II was designed with serviceability and easy mounting for all sensors as a primary concern. Hinging panels are used as access ports for the battery and tether connection to limit the number of fasteners needed to perform normal maintenance and service. If the outer shell needs to be opened, a single wrench is used and can be accomplished easily.

Additionally, rigid mounts are used for every component both for rigid sensor positions that allow the development of autonomous software, and for easy assembly of the system. The mounts ensure that all components are held tightly and have an index position that they can be aligned to through successive assembly and disassembly cycles, preventing software correction and calibration. Figure 6 shows one of the mounting assemblies used on vehicle, in this case for the imaging sonar and front camera.

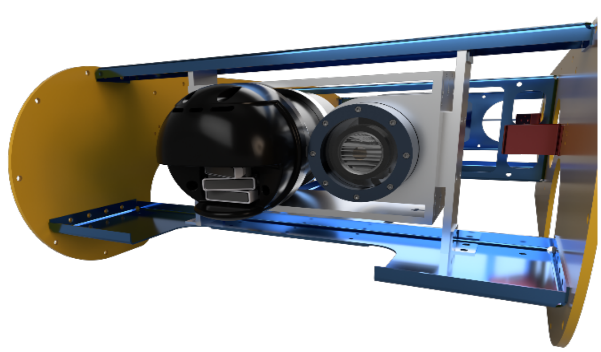

Figure 6: Front sensor mount

G. *Current Developments*

Yellowfinn II does not yet have systems for completing all of the tasks. Currently, the droppers and torpedo launchers are being developed.

The dropper system in development consists of small 3D-printed torpedoes that are released by a servo. These 3D-printed torpedoes are hollow so that they can be weighted to fall straight.

The torpedo system consists of the same 3D printed design, only without the weighting, and are spring launched and released by a servo. Having them spring launched ensures that they are released at a safe speed and won't cause injury to the pool or diver.

III. Electrical Subsystem

The systems of YellowFinn II consist of student made, donated, and purchased components. The electronics are designed and implemented to be compact and modular, making full use of creative design and novel means of connectivity. Particular attention was given to reliability, efficiency, and graceful fault-handling.

Much was accomplished through the use of printed circuit boards as communications and power backplanes, and as host boards to various modules. These boards contributed to the compact nature of the system by interfacing directly to the Glenair connectors via socketed contacts, eliminating flying leads and permitting rapid assembly and ease of maintenance while preserving the valuable connectors by eliminating solder and crimp contact usage.

A. *Computing*

The computing subsystem employs a heterogeneous architecture, mixing a CPU-focused Pico-ITX computer with two Jetson TX1 GPGPU platforms, shown in Figure 7. Each platform may directly retrieve necessary data from respective inlets via a fully meshed network, which also permits communication between platforms. A tabulated list of computing devices and their specifications are included in Table 1.

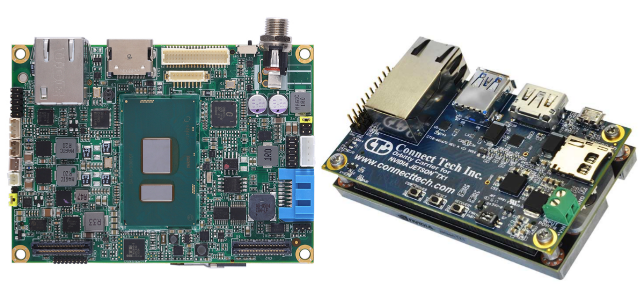

Figure 7: Jetson Computing Module on Carrier Board

Table 1: Computer Specifications

Pico-ITX

System	Axiomtek PICO500
Processor	Intel i7-6600U @ 2.6 GHz
Memory	16GB DDR4 SODIMM
Storage	256 GB Solid State
Jetson TX1 (x2)	
System	Connecttech Orbitty
Processor	Tegra X1: 4x A57 (big.LITTLE) + 256 CUDA cores
Memory	4GB LPDDR4
Storage	32GB eMMC

B. *Power*

The power supply originates from two paralleled yet physically separated custom 4 cell Lithium-Ion battery packs sponsored and manufactured by Aptus Aerospace LLC, which output a nominal 14.8V under load and provide a total capacity of 44Ah (652 Wh).

These batteries are continually monitored and protected by their first interface with the vehicle's power system: the battery cut-off board. This student-designed and manufactured board serves as a constant watchdog for over-current, over-/ under-voltage and leak fault detection. Short-lived or transient faults are handled gracefully by the dedicated hot-swap controller, which manipulates a rank of parallel MOSFETs to limit current (for transient faults) and ultimately protect the system from damage in the event of a persistent short or internal leak.

This system also implements an inrush current limiting routine on power-up, protecting connectors from damaging arcs during insertion, and preventing erroneous over-current faults. In addition, the board provides a digital reference for supply voltage, total system power, and other supply status matters. This information is provided via a robust and electrically isolated interface, and is utilized to gauge system health, utilization, and remaining mission time.

The power system is bisected into two fully isolated systems: Propulsion, which receives unregulated power from the cutoff board, and Logic, which receives power on multiple voltage rails derived by isolated DC-DC converters implemented on another student-designed and manufactured circuit board, shown in Figure 8. The voltage rails and the components they power are tabulated in Table 2. The purpose of this electrical isolation is to prevent the ground noise of the propulsion system from propagating to the more sensitive and delicate computing and sensory systems. This isolation is fully realized by the utilization of Ethernet communications between said systems, which by nature provides galvanic isolation.

Table 2: Voltage Rails

14.8V	Raw Battery Power: *Propulsion* Thrusters: fused, switched, monitored
5V	Isolated Logic Supply: Pico-ITX, Servos, Actuators Fused, monitored, regulated
12V	Isolated Logic Supply: *Jetsons, Pico-ITX, General* Fused, monitored, regulated
24V	Isolated Logic Supply: *Sonar, Auxiliary, Sensors* Fused, monitored, regulated
48V	Isolated Logic Supply: *Power over Ethernet* Fused, monitored, regulated

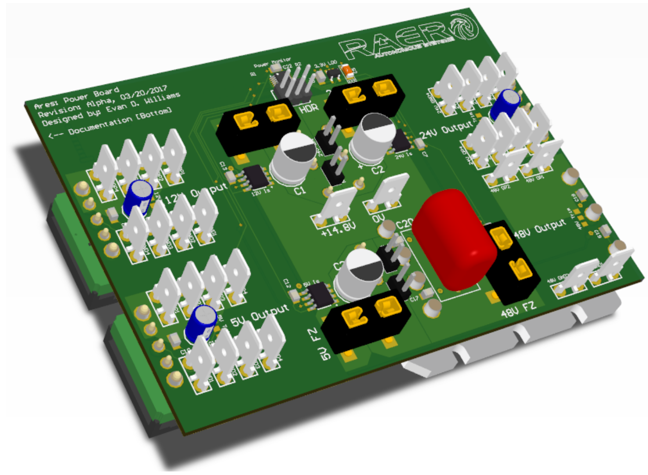

Figure 8: Power Board

C. *Navigation*

YellowFinn II uses dual VN-100 AHRS with mutual filtering as a primary navigation aid to determine headings. These AHRS units were generously donated to the Robotics Association at Embry-Riddle by VectorNav. The main features of this IMU are 0.02º RMS precision angular displacement measurements, simple 2D magnetic field calibration, and ability to automatically filter out soft and hard iron disturbances. These filtering capabilities are vital when operating in close proximity to large switching currents, such as the motor controllers.

The heading data from the AHRS is augmented by velocity and depth data from a Doppler Velocity Logger: the DVL-1000 by Nortek. Combining the highly accurate heading data of the AHRS with the precise velocity vectors of the DVL results in a robust position and attitude awareness with which to process data gathered from other sensors.

D. *Propulsion*

The propulsion system of the vehicle is implemented on a student designed and manufactured printed circuit board, which hosts seven electronic speed controllers organized into two pairs (laterals) and one set of three (verticals) in easily inspected and replaceable modules. The board implements power control for each set of ESCs, allowing for management of potential failure modes whilst permitting continued operation. The controllers are interface via simple PPM lines to the primary microcontroller of the system, a Teensy 3.6 running at 180MHz, with a shared and fully arbitrated serial line to retrieve telemetry data (current, voltage, RPM, status) from each controller. Commands and telemetry are communicated over an Ethernet interface with the rest of the vehicle network.

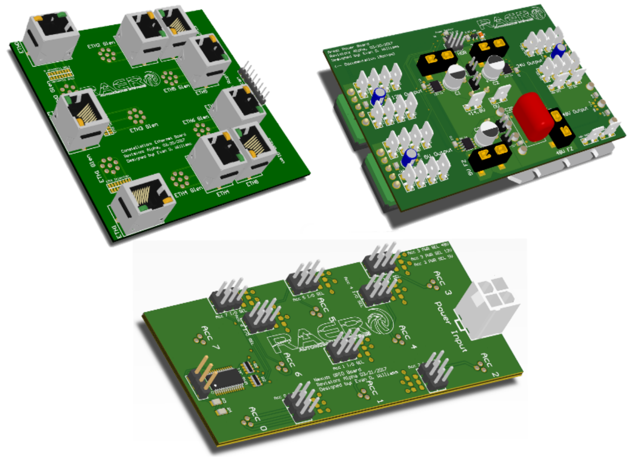

Figure 9: Assorted Custom Boards

E. Hydrophones

To accomplish acoustic localization, four Teledyne Reson TC4013 hydrophones are arranged into a diamond shaped ultra-short baseline array (USBL) with a spacing between the center of the sensors of 1.855 cm. This spacing was based on half the wavelength of the highest frequency in consideration, for our purposes 40 kHz. The mathematics of this sizing is included in the appendix for reference. This array is designed to extend from the front of the vehicle facing downwards. This array extends approximately a foot away from the vehicle so as to ensure that reflections are not immediate present off of the panels of the vehicle.

F. DAQ and Preamplifier

To amplify the signal from the hydrophones to a usable voltage an Analog Devices ADA4610-4 op-amp is used. This amplifier was selected because it gives 4 channels of amplification within one small format SMD chip, and has exceptional noise characteristics. This chip is well suited to the task because it offers low noise output for high impedance sensor amplification at high frequencies. An Analog Devices AD8054AR-EBZ evaluation board was used. This allowed for rapid development and easy prototyping. This board allows for configuration of analog signal processing. 19dB of gain was selected to achieve a signal at an appropriate voltage to be read by the DAQ. A NI cDAQ-9181 was selected to be the analog to digital converter. This was selected because it can perform 500 kilo samples per second, simultaneously with 16-bit resolution. This DAQ is connected via Ethernet to allow for high speed communications.

IV. SOFTWARE SUBSYSTEM

Based on the year over year performance of the vehicle, and the high impact of software on that performance, the methodology of software development was adapted for this year's vehicle. The goal of the design was two-fold, to enable development in a manner that best supported multiple simultaneous developers as well as stressed robust implementation. These focuses resulted in a software subsystem of the highest quality yet for the team. Rigorous testing of the modules throughout the development resulted in stable and bug-free (software) that was ready to be deployed on the vehicle upon completion of the new chassis.

The software architecture of Yellowfinn is built on a publish/subscribe style. In this implementation, core modules of the software are built as standalone modules and are linked through a network interface known as Minion Core, originally designed by our students for our RobotX platform. Minion Core manages the passing of messages between any piece of code networked with the vehicle, independent of physical computing device, or hard line wiring within the vehicle. This enabled students to easily develop individual pieces of code independently of one another, with only the required data format to be known for individual messages. The following sections will discuss the specific modules used within the platform and their functionality.

A. STATE

The first module in consideration is that of state. This module is responsible for determining all location and orientation parameters of the vehicle. This is accomplished through an Extended Kalman Filter between the AHRS/IMU and DVL, discussed prior. The goal of this filter is to not just accurately determine the speed of the vehicle, but also its position in space. This is imperative for getting the most out of the imaging sonar, as a global map of the environment can be created and persistently held.

B. CONTROLS

The controls module serves to calculate and command the necessary motor signals for the full 6 degree of freedom vehicle. The control system itself functions on multiple sets of cascaded PID control loops. For a given input (Roll, Pitch, Yaw, X, Y, Z) each of these inputs first goes through an outer layer PID. This serves to compute a desired rate in the case of the angles and speed in the case of the positions. These are then fed into an inner loop running at ten times the speed to prevent integrator windup. The actual model of the vehicle was calculated from a purely physical arrangement of motors and the center of rotation. This model outputs a desired thrust from the PIDs which can be mapped to the PWM signal to command the motors through a thrust curve.

C. VISION

The first of two sensing modules is that of vision. This module aggregates data produced from the two Nvidia Jetson modules on board and their respective cameras. To classify targets, the algorithm You Only Look Once was implemented with a competition specific dataset.[1] This algorithm uses a convolutional neural network to detect the desired objects we come across and locate them in frame. Performance of the algorithm with our training set yields an average deployed accuracy of 98% for a correct classification at an update rate of 6 frames per second. With the objects classified, the Jetsons return the bounding boxes and classification of each and the module computes the relative location of these objects in the camera frame for use in the perception module. A sample frame of old

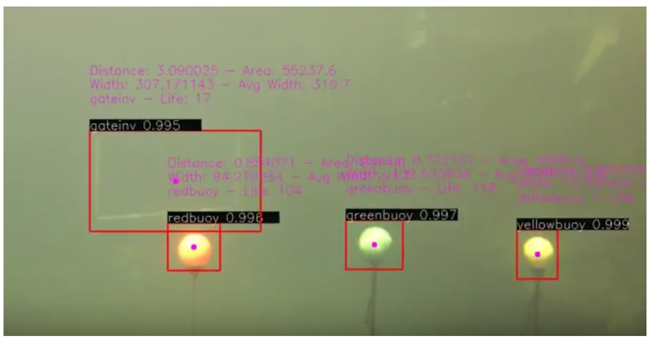

Figure 10: Buoy and Inverted Gate Classification

Transdec footage is shown in Figure 10 to illustrate how the detection bounding boxes surround the objects. Accuracy levels of over 99% are shown in the image as the objects are in clear water and relatively close, along with the buoys having the correct color identification, an issue seen in the testing of other algorithms.

D. PERCEPTION

The perception module serves to do two major operations. This first is to interface and process imaging sonar data. To do this, a 3-dimensional occupancy grid is created. Sonar returns are mapped into this occupancy grid. From this occupancy grid, classifications of objects can be made. This is done using an SVM classifier. Objects of interest are segmented and unique characteristics are determined in order to compute the classification. These unique characteristics form what is known as a feature vector. For our classification purposes, the features used are: Width, height, volume, and surface area. What this results in is a classifier that achieves 95+% accuracy. However, this is not the only piece of data the module uses in its determination. The classification from the vision module is compared with this data as both can be placed in the same reference frame. Not only does this serve to increase the accuracy of the classification, it also allows the color dependent classifications of the vision processing to help augment the lack of color data in the sonar.

E. PATH PLANNING

The path planning module serves to utilize the perception module data and determine the optimal path to the target location. To accomplish this, a 3-dimensional spatial implementation of A* is employed. The specific implementation uses heuristics determined through vehicle testing that account for the orientation and speed of the vehicle throughout the to optimize searching. The specific space it searches is the occupancy grid of the perception module with cells voxels possessing returns treated as obstacles. A representative path through a point cloud space is included for reference in Figure 11.

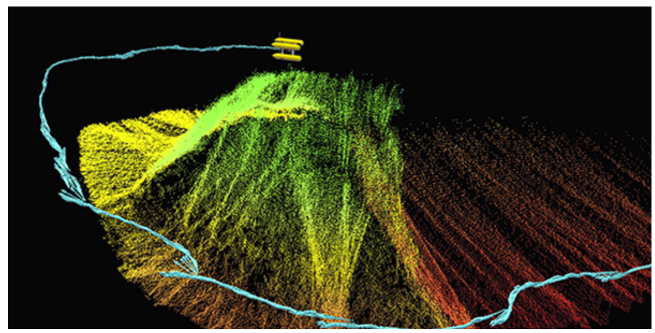

Figure 11: Representative path through point cloud. [2]

F. HYDROPHONES

The hydrophone module serves to interface with the DAQ and sensor array and compute the bearing to the sound source. The module uses a time difference of arrival solution to the determine the bearing based on a pure geometry solution to the array. The derivation of the mathematics is included in the appendices for reference. Beyond the mathematical solution, multiple other steps are taken into account to ensure that the correct bearing is calculated. Readings of multiple pulses are taken and the calculated bearings are placed into a histogram style binning of the results. After the desired number of readings is taken, the bin with the highest number of readings is taken and the values with averaged to be output as the bearing. This approach serves to filter out spurious readings as well as preventing multiple spurious readings from throwing off approaches that average all bearing computations. This methodology was validated at the Maritime RobotX Challenge with correct detections calculated. This served as a relevant test case it took place in an environment with large reflections of the sound source were also an issue much like we have seen in the Transdec in years past.

G. TASK MANAGER

The task manager module is the final primary component of the vehicle. It serves to aggregate the data other modules produce as well as its own data to make decisions as to what tasks to complete. Within the module, timing data of the current run and overall mission time is kept as a means of tracking the vehicles progress. It also serves as a metric for which task to complete, as the overall mission time runs low, high point value tasks are prioritized in order to maximize overall mission score.

To make these determinations, tasks are stored with quantifiable metrics to them. Each task not only has a maximum score associated with it, but also an expected score and an opportunity cost. The expected score serves to account for imperfections in the system (poor torpedo aiming, manipulation complexity, etc.). The opportunity cost considers how close a task is and weighs heavily on the decision until time runs low. The only exception to this is the qualifying gate, which is prioritized over everything else as it needs to be completed.

V. Conclusions

The goal of the team this year was to create a vehicle that was not just optimized for the competition but was a robust and proven platform overall. Through the design of all parts of the system discussed throughout this paper, the importance of robustness and safety was again and again brought up as this was crucial to meeting this design goal. The design approach of building robust sensing and capability instead of task specific capabilities proved also to help meet this goal for the platform.

Looking at the competition, the vehicle produced is the most capable and tested platform we have yet brought to the competition. It is a testament to the hard work of the students and their commitment to the success of the team. This hard work and dedication should be reflected in the performance of the vehicle this year as well as with the newfound knowledge and skills the team learned throughout the process.

VI. Acknowledgements

We would like to thank the numerous organizations for the support of the team and vehicle. Firstly, the Department of Electrical, Computer, Software and Systems Engineering on campus for their support of the project year in and year. Secondly, to the robotics research lab on campus for the access to the imaging sonar and dvl used on the vehicle. Thirdly, to the faculty advisors whose guidance was invaluable in throughout the entire process of the design and implementation of the vehicle. Finally, to our sponsors, namely GLENAIR, Volz, and ClickBond whose support was paramount to the development of the platform.

VII. References

1. Redmon, J., Divvala, S., Girshick, R., & Farhadi, A. (2016). You Only Look Once: Unified, Real-Time Object Detection. 2016 IEEE Conference on Computer Vision and Pattern Recognition (CVPR). doi:10.1109/cvpr.2016.91
2. Galceran, E. (2014). Coverage Path Planning for Autonomous Underwater Vehicles. Retrieved June 19, 2017.
3. Johnson, M., & Waterman, A. (2006). APLS - An Acoustic Pinger Location System For Autonomous Underwater Vehicles.

VIII. APPENDIX

A. Derivation of Planar Localization

The calculation used in this algorithm looks at a grouping of three hydrophones in a right triangle. The side length of the two shorter sides is the 1.855 cm, calculated in Equation 1. For the derivation of the solution, this will be defined as d, as this solution is for the general form of this array. Figure 10 shows the triangle and includes references to the hydrophones that will be used throughout the rest of the calculations. The goal of the calculations is to determine θ, the heading of the sound source relative to the triangular array.

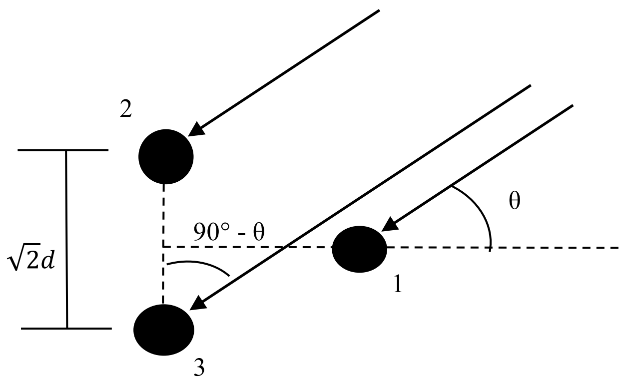

Figure 13: Hydrophone Array Geometry

The reference hydrophone is defined as 1. Based, on this there is a lag in the time it takes to reach hydrophone 2 and 3.

From **Error! Reference source not found.**1, a geometric relationship between θ and d can be established. Pairing this with the relationship between the speed of sound in a fluid, c_0, the time of arrival t_2 and t_3, the geometry of the array arrives at the geometric relationships of Equations 2, 3 and 4.

$$\cos(90° - \theta) = \sin(\theta) \tag{2}$$

$$\sin(\theta) = \frac{c_0(t_3 - t_2)}{\sqrt{2}d} \tag{3}$$

$$\cos(\theta - 45°) = \frac{c_0(t_3 - t_1)}{d} \tag{4}$$

Dividing Equation 4 by 3:

$$\frac{\cos(\theta - 45°)}{\sin(\theta)} = \frac{\frac{c_0(t_3 - t_1)}{d}}{\frac{c_0(t_3 - t_2)}{\sqrt{2}d}} = \sqrt{2}\frac{(t_3 - t_1)}{(t_3 - t_2)} \tag{5}$$

Using properties of cosine, we are can expand:

$$\cos(\theta - 45°) = \cos(\theta)\cos(45°) + \sin(\theta)\sin(45°) \tag{6}$$
$$= \cos(\theta)\frac{1}{\sqrt{2}} + \sin(\theta)\frac{1}{\sqrt{2}}$$

Dividing through by sin(θ) and simplifying:

$$\frac{\cos(\theta - 45°)}{\sin(\theta)} = \frac{\cos(\theta)}{\sin(\theta)}\frac{1}{\sqrt{2}} + \frac{\sin(\theta)}{\sin(\theta)}\frac{1}{\sqrt{2}} \tag{7}$$

$$\cot(\theta)\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}\frac{(t_3 - t_1)}{(t_3 - t_2)} = \frac{\sqrt{2}t_{31}}{t_{32}} \tag{8}$$

Simplifying the definition of time difference for the rest of the calculations:

$$t_{31} = t_3 - t_1 \tag{9}$$

Substituting (9) into (8) and simplifying:

$$\cot(\theta) + 1 = 2\frac{t_{31}}{t_{32}} \tag{10}$$

Solving for cot(θ):

$$\cot(\theta) = \frac{2t_{31}}{t_{32}} - \frac{t_{32}}{t_{32}} = \frac{2t_{31} - t_{32}}{t_{32}} \tag{11}$$

Solving for θ:

$$\theta = \tan^{-1}\frac{t_{32}}{2t_{31} - t_{32}} \tag{12}$$

Verification checks:
If θ = 0°, $t_{32} = 0$
If θ = 45°, $t_{32} = t_{31}$

Putting t in terms of ϕ, the phase shift, where f is the frequency of the signal:

$$t_{32} = \frac{2\pi(\phi_3 - \phi_2)}{360°\, f} \tag{13}$$

Substituting and simplifying:

$$\theta = \tan^{-1}\frac{\frac{2\pi(\phi_3 - \phi_2)}{360°\, f}}{2\frac{2\pi(\phi_3 - \phi_1)}{360°\, f} - \frac{2\pi(\phi_3 - \phi_2)}{360°\, f}}$$
$$= \tan^{-1}\frac{\phi_{32}}{2\phi_{31} - \phi_{32}} \tag{14}$$

This final simplification is used to speed up computation time. It was determined to be faster computationally to compute phase difference than time difference and as there is a direct correlation, the ratio is the same.